IMAGE SEGMENTATION USING CLUSTERING WITH SADDLE POINT DETECTION

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Abstract

We discuss a novel statistical framework for image segmentation based on nonparametric clustering. By employing the mean shift procedure for analysis, image regions are identified as clusters in the joint color-spatial domain. To measure the significance of each cluster we use a test statistics that compares the estimated density of the cluster mode with the estimated density on the cluster boundary. The cluster boundary in the color domain is defined by saddle points lying on the cluster borders defined in the spatial domain. The proposed technique compares favorably to other segmentation methods described in literature.

1. INTRODUCTION

Segmentation using clustering involves the search for image points that are similar enough to be grouped together. Algorithms from graph theory [4, 7], matrix factorization [13, 14], deterministic annealing [8], scale-space theory [9], and mixture models [5, 12] were successfully used to delineate relevant structures within the input data.

In this paper we present a new and practical approach to image segmentation using a nonparametric model for image regions. According to this model, the regions are seen as clusters associated to local maxima (modes) of the probability density function computed in the joint color-spatial domain [1]. To evaluate the cluster significance we employ a test statistic that compares the estimated density of the mode with the estimated density on the cluster boundary. The latter density is measured in the saddle points lying on the cluster border defined in the spatial domain. The detection of saddle points is based on a recently introduced algorithm [2].

The paper is organized as follows. Section 2 introduces the mean shift based data decomposition. Section 3 shortly describes the algorithm for saddle point detection and discusses the test statistic. Experimental segmentation results and comparisons are shown in Section 4.

2. MEAN SHIFT BASED DATA DECOMPOSITION

Given n data points $\mathbf{x}_i$, $i = 1 \ldots n$ in the d-dimensional space R^d, the multivariate mean shift vector computed with kernel K in the point $\mathbf{x}$ is given by [6, 1]

$$\mathbf{m}_K(\mathbf{x}) \equiv \frac{\sum_{i=1}^n \mathbf{x}_i K\left(\frac{\mathbf{x}-\mathbf{x}_i}{h}\right)}{\sum_{i=1}^n K\left(\frac{\mathbf{x}-\mathbf{x}_i}{h}\right)} - \mathbf{x}\,, \qquad (1)$$

where h is the kernel bandwidth. In the following we will use the symmetric normal kernel. It can be shown that the mean shift vector at location $\mathbf{x}$ is proportional to the normalized density gradient estimate computed with kernel K

$$\mathbf{m}_K(\mathbf{x}) = h^2 \frac{\hat{\nabla} f_K(\mathbf{x})}{\hat{f}_K(\mathbf{x})}. \qquad (2)$$

The normalization is by the density estimate in $\mathbf{x}$ obtained with kernel K. This formula changes a bit for kernels different from the normal [1].

The *mean shift procedure* is obtained by successive computation of the mean shift vector $\mathbf{m}_K(\mathbf{x})$ and translation of the kernel $K(\mathbf{x})$ by $\mathbf{m}_K(\mathbf{x})$. The procedure and is guaranteed to converge at a nearby point where the density estimate has zero gradient. A decomposition algorithm based on the mean shift property is discussed in [1].

3. SADDLE POINT DETECTION

We recently introduced in [2] an algorithm for finding the saddle points associated with a given bandwidth h and a partition $\{\mathbf{D}_u\}_{u=1\ldots m}$ obtained through mean shift decomposition. A short description of the algorithm is given in the sequel. First order saddle points are detected, having the Hessian matrix with one positive eigenvalue and all other eigenvalues negative.

Let us select a cluster index v and define the complementary cluster set

$$\mathbf{C}_v \equiv \bigcup_{u \neq v} \mathbf{D}_u. \qquad (3)$$

In the following we drop the index v for the simplicity of the equations. We define two functions

$$\hat{f}_{D,K}(\mathbf{x}) = \frac{1}{nh^d} \sum_{\mathbf{x}_D \in D} K\left(\frac{\mathbf{x} - \mathbf{x}_D}{h}\right) \qquad (4)$$

and

$$\hat{f}_{C,K}(\mathbf{x}) = \frac{1}{nh^d} \sum_{\mathbf{x}_C \in C} K\left(\frac{\mathbf{x} - \mathbf{x}_C}{h}\right) \qquad (5)$$

whose superposition at $\mathbf{x}$ equals the density estimate at $\mathbf{x}$

$$\hat{f}_K(\mathbf{x}) \equiv \frac{1}{nh^d}\sum_{i=1}^{n} K\left(\frac{\mathbf{x}-\mathbf{x}_i}{h}\right) = \hat{f}_{D,K}(\mathbf{x}) + \hat{f}_{C,K}(\mathbf{x}). \quad (6)$$

Computing now the gradient of expression (6), multiplying by h^2, and normalizing by $\hat{f}_K$ it results that

$$\mathbf{m}_K(\mathbf{x}) = h^2\frac{\hat{\nabla} f_K(\mathbf{x})}{\hat{f}_K(\mathbf{x})} = \alpha_D(\mathbf{x})\mathbf{m}_{D,K}(\mathbf{x}) + \alpha_C(\mathbf{x})\mathbf{m}_{C,K}(\mathbf{x}) \quad (7)$$

where

$$\mathbf{m}_{D,K}(\mathbf{x}) = \frac{\sum_{\mathbf{x}_D\in D}\mathbf{x}_D K\left(\frac{\mathbf{x}-\mathbf{x}_D}{h}\right)}{\sum_{\mathbf{x}_D\in D} K\left(\frac{\mathbf{x}-\mathbf{x}_D}{h}\right)} - \mathbf{x} \quad (8)$$

$$\mathbf{m}_{C,K}(\mathbf{x}) = \frac{\sum_{\mathbf{x}_C\in C}\mathbf{x}_C K\left(\frac{\mathbf{x}-\mathbf{x}_C}{h}\right)}{\sum_{\mathbf{x}_C\in C} K\left(\frac{\mathbf{x}-\mathbf{x}_C}{h}\right)} - \mathbf{x} \quad (9)$$

are the mean shift vectors computed only within the sets $\mathbf{D}$ and $\mathbf{C}$ respectively, and

$$\alpha_D(\mathbf{x}) = \frac{\hat{f}_{D,K}(\mathbf{x})}{\hat{f}_K(\mathbf{x})} \qquad \alpha_C(\mathbf{x}) = \frac{\hat{f}_{C,K}(\mathbf{x})}{\hat{f}_K(\mathbf{x})} \quad (10)$$

with $\alpha_D(\mathbf{x}) + \alpha_C(\mathbf{x}) = 1$. Equation (7) shows that the mean shift vector at any point $\mathbf{x}$ is a weighted sum of the mean shift vectors computed separately for the points in the sets $\mathbf{D}$ and $\mathbf{C}$.

We exploit this property for the finding of saddle points. Assume that $\mathbf{x}_s$ is a saddle point of first order located on the boundary between $\mathbf{D}$ and $\mathbf{C}$. The boundary condition is

$$\mathbf{m}_K(\mathbf{x}_s) = \mathbf{0} \quad (11)$$

which means that the vectors $\alpha_D(\mathbf{x}_s)\mathbf{m}_{D,K}(\mathbf{x}_s)$ and $\alpha_C(\mathbf{x}_s)\mathbf{m}_{C,K}(\mathbf{x}_s)$ have equal magnitude, are collinear, but point towards opposite directions.

Let us define the vectors

$$\mathbf{r}_D(\mathbf{x}) = \frac{\|\alpha_C(\mathbf{x})\mathbf{m}_{C,K}(\mathbf{x})\|}{\|\alpha_D(\mathbf{x})\mathbf{m}_{D,K}(\mathbf{x})\|}\alpha_D(\mathbf{x})\mathbf{m}_{D,K}(\mathbf{x}) \quad (12)$$

and

$$\mathbf{r}_C(\mathbf{x}) = \frac{\|\alpha_D(\mathbf{x})\mathbf{m}_{D,K}(\mathbf{x})\|}{\|\alpha_C(\mathbf{x})\mathbf{m}_{C,K}(\mathbf{x})\|}\alpha_C(\mathbf{x})\mathbf{m}_{C,K}(\mathbf{x}) \quad (13)$$

obtained by switching the norms of $\alpha_D(\mathbf{x}_s)\mathbf{m}_{D,K}(\mathbf{x}_s)$ and $\alpha_C(\mathbf{x}_s)\mathbf{m}_{C,K}(\mathbf{x}_s)$. Note that in case of a perturbation of $\mathbf{x}_s$ towards $\mathbf{C}$ and along the line defined by $\alpha_D(\mathbf{x}_s)\mathbf{m}_{D,K}(\mathbf{x}_s)$ and $\alpha_C(\mathbf{x}_s)\mathbf{m}_{C,K}(\mathbf{x}_s)$, the resultant

$$\mathbf{r}(\mathbf{x}) = \mathbf{r}_D(\mathbf{x}) + \mathbf{r}_C(\mathbf{x}) \quad (14)$$

will point towards the saddle point. Since the saddle point is of first order, it will be also stable for the directions perpendicular to $\mathbf{r}(x)$ hence it will be a stable point with basin of attraction.

The algorithm uses the newly defined basin of attraction to converge to the saddle point. The saddle point detection should be started close to a valley, i.e., at locations having divergent mean shift vectors coming from the sets $\mathbf{D}$ and $\mathbf{C}$

$$\alpha_D(\mathbf{x})\alpha_C(\mathbf{x})\mathbf{m}_{D,K}(\mathbf{x})^\top\mathbf{m}_{C,K}(\mathbf{x}) < 0 \quad (15)$$

Since the data is already partitioned it is simple to search for points that verify condition (15). If one starts the search from a point in $\mathbf{D}$ just follow the mean shift path defined by $\mathbf{m}_{C,K}(\mathbf{x})$ till the condition (15) is satisfied. Nevertheless, if the cluster $\mathbf{D}$ is isolated, the function $\hat{f}_{C,K}(\mathbf{x})$ (5) will be close to zero for the data points belonging to $\mathbf{x} \in \mathbf{D}$ and can generate numerical instability. Therefore a threshold should be imposed on this function before computing $\mathbf{m}_{C,K}(\mathbf{x})$. The algorithm for finding the saddle points lying on the border of $\mathbf{D}$ is given below.

Saddle Point Detection

Given a data partitioning into a cluster $\mathbf{D}$ and another set $\mathbf{C}$ containing the rest of the data points. For each $\mathbf{x}_D \in \mathbf{D}$, if the value of $\hat{f}_{C,K}(\mathbf{x}_D)$ (5) is larger than a threshold

1. Follow the mean shift path defined by $\mathbf{m}_{C,K}(\mathbf{x})$ (9) until the condition (15) is satisfied.
2. Follow the mean shift path defined by $\mathbf{r}(\mathbf{x})$ (14) until convergence.

Denote by $\mathbf{x}_s$ the saddle point with the largest density lying of the border of a given cluster characterized by the mode $\mathbf{y}_m$. The point $\mathbf{x}_s$ represents the "weakest" point of the cluster border. It requires the least amount of probability mass which should be taken from the neighborhood of $\mathbf{y}_m$ and placed in the neighborhood of $\mathbf{x}_s$ such that the cluster mode disappears, as described in [11].

In [2] we have derived the test statistic for the null hypothesis of the mode existence

$$z = \frac{\sqrt{n_c}}{2}\frac{\hat{f}_K(\mathbf{y}_m) - \hat{f}_K(\mathbf{x}_s)}{\sqrt{\hat{f}_K(\mathbf{y}_m)\hat{f}_K(\mathbf{x}_s)}} \quad (16)$$

where $\hat{f}_K(\mathbf{y}_m)$ is the probability density at the mode location and $\hat{f}_K(\mathbf{x}_s)$ is the density at $\mathbf{x}_s$. The p-value of the test is the probability that z, which is distributed with $N(0,1)$, is positive

$$\text{Prob}(z \geq 0) = \frac{1}{\sqrt{2\pi}}\int_{-z}^{\infty}\exp(-t^2/2)dt \quad (17)$$

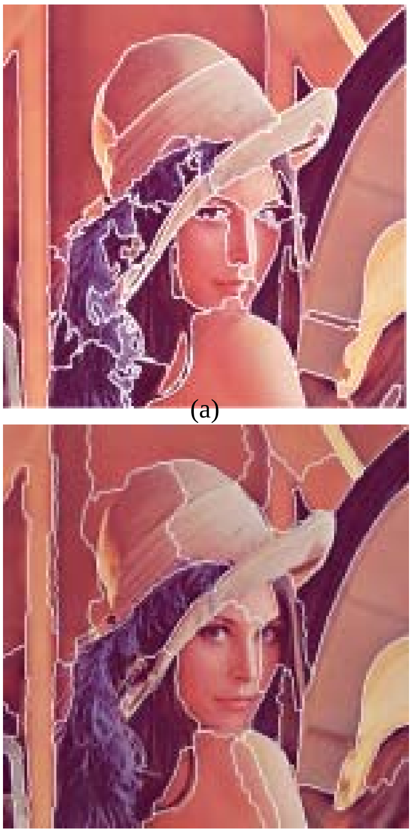
A confidence of 0.95 is achieved when $z = 1.65$.

To test the existence of two neighboring clusters, we adapt the test by replacing $\hat{f}_K(\mathbf{y}_m)$ by $\frac{\hat{f}_K(\mathbf{y}_1)+\hat{f}_K(\mathbf{y}_2)}{2}$ where $\hat{f}_K(\mathbf{y}_1)$ and $\hat{f}_K(\mathbf{y}_2)$ are the densities associated to the modes of the two clusters. In this case, $\mathbf{x}_s$ is taken as the common saddle point with the largest density.

4. SEGMENTATION EXPERIMENTS

We adapt the framework presented in Section 3 for the characterization of image clusters in the joint color-spatial domain. The idea is to start with a given decomposition (over-segmentation) and join the least significant clusters until they become significant according to the measure (17). We use the image segmentation framework described in [1], which employs mean shift to delineate clusters in a joint space of dimension $d = r + 2$ that includes the spatial coordinates ($r = 3$ for color images). All experiments presented here are performed with a bandwidth $h_r = 20$ for the color information, and $h_s = 4$ for the spatial domain. To characterize the joint-domain clusters, we run the saddle point detection algorithm for each pixel on the cluster boundary. However, the spatial component is fixed and only the color component varies. Then, for every pair of two neighboring clusters we compute the mean density associated with their borders and their peak densities. These values are used in (17) to determine the significance of the cluster pair. Only clusters with confidence larger than 0.9 are retained.

Figure 1 presents the segmentation of image *Lenna* using the new method. We compare our results with the a recent segmentation method described in [3]. The two algorithms have roughly the same complexity (a few seconds on a typical PC for 512×512 images). Observe the quality of hat delineation in comparison to the technique [3]. Two sets of segmented images using the same parameters are presented in Figure 2 and Figure 3 for data coming from Berkeley database [10]. Observe the high quality of contours.



(a)

(b)

Fig. 1. Contours for *Lenna*. (a) Our method. (b) Method [3].

5. CONCLUSION

This paper shows that hypothesis testing for segmentation is an effective direction for solving decomposition problems and evaluating the significance of the results.

Acknowledgement

We acknowledge the use of test images from Berkeley database [10].

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Fig. 2. Segmentation of images from Berkeley database.

Fig. 3. Segmentation of images from Berkeley database.